ERROR ESTIMATION IN PLANT GROWTH ANALYSIS

ANDRZEJ GREGORCZYK

Department of Plant Physiology, Agricultural University in Szczecin
Słowackiego 17, 71-434 Szczecin, Poland

(Received: July 14, 1995. Accepted: May 31, 1996)

ABSTRACT

The scheme is presented for calculation of errors of dry matter values which occur during approximation of data with growth curves, determined by the analytical method (logistic function) and by the numerical method (Richards function). Further formulae are shown, which describe absolute errors of growth characteristics: Growth rate (GR), Relative growth rate (RGR), Unit leaf rate (ULR) and Leaf area ratio (LAR). Calculation examples concerning the growth course of oats and maize plants are given. The critical analysis of the estimation of obtained results has been done. The purposefulness of joint application of statistical methods and error calculus in plant growth analysis has been ascertained.

KEY WORDS: plant growth analysis, dry matter accumulation, error calculus, regression functions, *Zea mays* L., *Avena sativa* L.

INTRODUCTION

Observation of development and measurements of plant growth are an important source of knowledge on life processes of vegetal organisms. Mathematical analysis of plant growth is a valuable method of working out the results of measurements and for dozens of years it was used in botanical sciences. In this method the classical growth analysis and functional approach can be distinguished. Classical analysis affords possibilities for examining the plant dry matter increase in definite harvest intervals, and it also allows to define the assimilates distribution between assimilating and nonassimilating parts of plant organism. The functional approach, generally speaking, induces representation in graphical form of growth measurements (expressed e. g. as dry matter, height) as a function of time. In order to attain a more precise usage of information comprised in plant growth analysis procedures it seems that apart from statistical methods the error calculus should be applied. Statistical methods provide possibilities of evaluation of significance of specific growth model (using parametric or nonparametric test), they also enable the estimation, of a confidence interval for the result. Another important problem is the estimation, of an error which can be committed during calculation of plant growth characteristics. Error analysis facilitates the critical evaluation of the obtained results, especially in case of functions of several variables. The statistical methods in error calculus are very often used, e. g. in order to calculate the confidence interval for arithmetic mean by Student's t-test. In general one meets errors of a random character which result from the random distribution of a given variable and errors resulting of the calculation algorithm which was accepted.

Vegetation time can be considered as a real variable, because it is defined directly with a sufficient precision. But the biometric quantities such as: plant height, dry matter, leaf blade area must be treated as random variables with normal distribution or close to the normal one.

ERRORS MADE AT APPROXIMATION
OF DATA WITH GROWTH CURVES DETERMINED
BY ANALYTICAL METHOD

In practice most often measurements are taken of a given quantity, e. g. dry matter W_i in particular plant growth stages or in identical time intervals t_i ($i = 1, 2, \dots, n$). Finally, one obtains ordered sequences of numbers $\{W_i, t_i\}$ which are generally called time series. The problem of growth curve determination consists in finding regression function $W = f(t)$. Numerous growth models (functions) can be found in literature (Causton and Venus 1987, Evans 1972, Hunt 1982, Żelawski and Lech 1980), these are first of all solutions of nonlinear differential equations of the 1st order. Typical growth curves have the so-called sigmoid shape, similar to the horizontally developed letter S.

The most common growth functions undergo linearization by logarithmic transformation – therefore it is possible to apply for them the standard regression procedure of the method of least squares. This procedure provides possibilities for calculation of two independent growth curve parameters, which are random variables (Bliss 1970, Chatterjee and Price 1977, Weisberg 1985).

Great importance in plant growth analysis has found the logistic function in the form:

$$W = W_{\max} / (1 + b e^{-kt}) \quad (1)$$

where:

- W – dry matter,
t – time,
W_{max} – asymptotic value of final size
b, k – constant parameters (b>0, k>0).

Below, the scheme of calculation of confidence intervals for the logistic curve, with earlier assigned significance level is presented. The discussed case refers to the situation, when for every t value one W value corresponds. Simultaneously it is assumed that the measured W_i values are loaded with the statistical error ε_i which is a random variable with expected zero value [E(ε_i) = 0] and normal distribution [N(0, σ)]. Moreover ε errors for different measurements are mutually uncorrelated [cov(ε_i, ε_j) = 0; i ≠ j] and t_i values are unbiased with statistical errors (Snedecor and Cochran 1968, Steel and Torrie 1987).

After logarithmic transformation of equation (1) one gets the dependence:

$$\ln(W_{\max} / W_i - 1) = \ln b - kt_i, \quad (2)$$

which corresponds to the linear function:

$$Y_i = A_0 + B t_i. \quad (3)$$

Then the standard error of arbitrary estimation of average value $\hat{Y}_i$, $S_{\hat{Y}_i}$ should be calculated:

$$S_{\hat{Y}_i} = \sqrt{S_E^2/n + S_B^2(t_i - \bar{t})^2} \quad (4)$$

where:

- n – number of observations
 $\bar{t}$ – arithmetic mean of time value
S_B – standard error of regression coefficient
S_E – standard error of estimation given by equation:

$$S_E = \sqrt{\frac{\sum_{i=1}^n (Y_i - \hat{Y}_i)^2}{n-2}} \quad (5)$$

where:

- Y_i – empirical values
 $\hat{Y}_i$ – theoretical values.

Confidence limits (ΔY_i) for Y_i are determined from equation:

$$\Delta Y_i = S_{\hat{Y}_i} \cdot t_{\alpha, n-2} \quad (6)$$

where:

- t_{α, n-2} – critical value of Student's distribution for significant level α with (n-2) degrees of freedom read from tables, in which P(|t| ≥ t_α) = α.

This way the confidence band for linear regression function (3) can be obtained. Now, the confidence limit for $\hat{W}_i$, value should be calculated. To attain this the equation:

$$Y_i = \ln(W_{\max} / W_i - 1) \quad (7)$$

should be first transformed in relation to W_i:

$$W_i = W_{\max} / [1 + \exp(Y_i)]. \quad (8)$$

Then using simplified Taylor series for function y = f(x) (Korn and Korn 1968):

$$y \pm \Delta y = f(x) + \frac{df(x)}{dx} \Delta x, \quad (9)$$

the absolute error ΔW_i is calculated:

$$\Delta W_i = \left| \frac{df}{dY_i} \right| \Delta Y_i = \frac{W_{\max} \exp(Y_i)}{[1 + \exp(Y_i)]^2} \cdot \Delta Y_i. \quad (10)$$

As an example for calculation of errors for growth curve determination by means of the analytical method the numerical data of measurements of dry matter accumulation in the maize plant (*Zea mays* L.) were used. These data are compiled in Table 1.

TABLE 1. Experimental and theoretical values of dry matter in maize plants (*Zea mays* L.).

Measurement, i	Time, t _i (day)	Empirical value, W _i (g)	Y _i = ln(145.62/W _i - 1)	Theoretical value, $\hat{W}_i$ (g)
1	48	3.08	3.835	2.839
2	69	15.79	2.107	18.798
3	85	69.81	0.0825	59.190
4	98	94.01	-0.5997	102.47
5	125	141.48	-3.5315	141.13
6	142	145.62	-∞	144.71

For linear regression analysis only 5 measurements were taken, because for W=W_{max} the value Y is indefinite:

$$\lim_{W \rightarrow W_{\max}} Y = \lim_{W \rightarrow W_{\max}} \ln(W_{\max} / W - 1) = -\infty \quad (11)$$

Calculations were done according to the algorithm showed above (equations 2-10). After simple calculations the form of linear function has been obtained:

$$\hat{Y}_i = 8.50911 - 0.0956534 t_i \quad (12)$$

and the determination coefficient r² = 0.9932**.

The real form of the logistic function has also been obtained, approximating the increase of dry matter in maize plants:

$$\hat{W}_i = \frac{145.62}{1 + 4.95974 \cdot 10^3 \exp(-9.56534 \cdot 10^{-2} \cdot t_i)} \quad (13)$$

Then according to equations (4) and (6), after finding in Student's-t distribution tables t_{0.05;5} = 3.182 value, the 95% confidence limits for function (12) were reckoned:

$$\begin{aligned} \Delta \hat{Y}_i &= 3.182 \sqrt{0.265272^2/5 + (4.5534 \cdot 10^{-3})^2(t_i - 85)^2} = \\ &= 3.182 \sqrt{0.0140738 + 0.207334 \cdot 10^{-4}(t_i - 85)^2}. \end{aligned}$$

Regression line (12) and confidence band are presented in Fig. 1.

The absolute error was calculated as an example for the second measurement (i = 2) from Table 1 (t_i = 69 day; $\hat{W}_i$ = 18,798 g).

$$\hat{Y} = 8.50911 - 0.0956534 \cdot 69 = 1.90902 \approx 1.909$$

Fig. 1. Regression line with 95% confidence band (explanations – see text).

$$\Delta Y = 3.182 \sqrt{0.0140738 + 0.207334 \cdot 10^{-4} (69 - 85)^2} = 0.44299 \approx 0.443$$

$$Y = 1.909 \pm 0.443$$

$$\Delta W = \frac{145.62 \exp(1.90902)}{[1 + \exp(1.90902)]^2} \cdot 0.44299 = 7.25243 \approx 7.252$$

$$W = \hat{W}_i \pm \Delta W$$

$$W = 18.798 \pm 7.252$$

It should be noticed that the estimation error is an independent variable function and achieves its minimum for $t_i = \bar{t}$.

ERRORS IN CALCULATION OF GROWTH CURVE PARAMETERS BY NUMERICAL METHODS

Values of the two most independent parameters of the searched growth function can be calculated by the standard linear regression method. Estimation of a greater number of growth curve parameters is possible by nonlinear regression methods. One of the most frequently used methods is the Marquardt's algorithm (Dreper and Smith 1973), which, however, requires

an initial estimation of the wanted parameters. Growth function coefficients are calculated numerically with the determined standard error, which can be used for confidence interval construction for each parameter. To attain this the Student's-t distribution with degrees of freedom equal to observation number diminished by the number of parameters may be used. It should be mentioned that nonlinear estimation methods are approximating methods, frequently basing on linear regression equations. Therefore the exact determination of the confidence band for a growth curve or confidence region for all estimated parameters is hindered or even impossible (Draper and Smith 1973). For an approximated estimation of maximal error value the simplified form of the law of variance propagation (Kvet et al. 1971) can be used. If describing quantity (e.g. dry matter) is a function of many variables: $y = f(x_1, x_2, \dots, x_N)$, then the variance of that variable s_y^2 is shown as an equation:

$$s_y^2 = \sum_{i=1}^N \left(\frac{\partial f}{\partial x_i} \right)^2 s_{x_i}^2 \quad (14)$$

where $s_{x_i}^2$ is a variance i of this independent variable.

The variance calculated in such a way is characterized by scattering of estimated y values in relation to regression equation.

A frequently applied function in growth analysis is the Richard's function:

$$W = W_{\max} (1 + b e^{-kt})^c \quad (15)$$

The meanings of symbols: W , $W_{\max}$, b , k is identical as in equation (1); but c is an additional constant coefficient.

Richard's curve is a function of 4 independently calculated coefficients: $W = f(W_{\max}, b, k, c)$. In that case basing on equation (14) the variance s_W^2 is:

$$s_W^2 = \left(\frac{\partial f}{\partial W_{\max}} \right)^2 s_{W_{\max}}^2 + \left(\frac{\partial f}{\partial b} \right)^2 s_b^2 + \left(\frac{\partial f}{\partial k} \right)^2 s_k^2 + \left(\frac{\partial f}{\partial c} \right)^2 s_c^2 \quad (16)$$

In the work of Gregorczyk (1991), basing on 12 measurements of dry matter accumulation in oat plant (*Avena sativa* L.), this process has been approximated by the logistic function in the form:

$$\hat{W}(t) = \frac{24.8}{1 + 240.1 \exp(-0.1331 t)} \quad (17)$$

The applied regression procedure proved to be effective: determination coefficient – $R^2 = 0.9939^{**}$ ($r_{0.01;12-2} = 0.708$).

Values of logistic function coefficients (17) were used in the present work for finding the constant parameters of Richard's function (15) by Marquardt's method (Gregorczyk 1994). Numerical calculations were done using STATGRAPHICS programme. After 5 iterations the Richard's function was obtained:

$$\hat{W} = 25.6187 [1 + 184.074 \exp(-0.11929 t)]^{-0.921622} \quad (18)$$

and standard errors of parameters: $s_{W_{\max}} = 0.61712$, $s_b = 257.824$, $s_k = 0.021319$, $s_c = 0.331366$.

The Richard's function describes the process of dry matter accumulation in oat plants more precisely ($R^2 = 0.9983^{**}$) than the logistic function. Fig. 2 shows the Richard's curve (equation 18) against a background of experimental points.

For variance estimation according to equation (16) the measurement performed at $t = 50$ day of oat vegetation has been selected. In this definite case the values of particular partial derivatives are as follows:

$$\frac{\partial f}{\partial W_{\max}} = (1 + b e^{-kt})^c = [1 + 184.074 \exp(-0.11929 \cdot 50)]^{-0.921622} = 0.69991$$

$$\begin{aligned} \frac{\partial f}{\partial b} &= W_{\max} c (1 + b e^{-kt})^{c-1} e^{-kt} = \\ &= 25.6187 \cdot (-0.921622) [1 + 184.074 \exp(-0.11929 \cdot 50)]^{-1.921622} e^{-0.11929 \cdot 50} = -0.028819 \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial k} &= W_{\max} c b (-t) e^{-kt} (1 + b e^{-kt})^{c-1} = \\ &= 25.6187 \cdot (-0.921622) \cdot 184.074 \cdot (-50) e^{-0.11929 \cdot 50} [1 + 184.074 \exp(-0.11929 \cdot 50)]^{-1.921622} = 265.24 \end{aligned}$$

$$\begin{aligned} \frac{\partial f}{\partial c} &= W_{\max} (1 + b e^{-kt})^c \ln(1 + b e^{-kt}) = \\ &= 25.6187 [1 + 184.074 \exp(-0.11929 \cdot 50)]^{-0.921622} \cdot \ln[1 + 184.074 \exp(-0.11929 \cdot 50)] = 6.9418 \end{aligned}$$

According to equation (16) variance $s_{\hat{W}}^2$ equals:

$$s_{\hat{W}}^2 = 0.69991^2 \cdot 0.61712^2 + (-0.028819)^2 \cdot 257.824^2 + 265.24^2 \cdot 0.021319^2 + 6.9418^2 \cdot 0.33137^2 = 92.661,$$

and standard deviation $s_{\hat{W}} = \sqrt{92.661} = 9.626$.

Fig. 2. Richard's curve fitted to experimental data of dry matter accumulation of oat plant (*Avena sativa* L.).

Standard deviation $s_{\hat{W}}$ has the meaning of the mean square error.

After finding in tables the critical value for Student's-t distribution, $t_{0.05; 12.4} = 2.306$, the 95% confidence intervals for all Richard's function parameters can be estimated:

$$W_{\max} = 25.6187 \pm 2.306 \cdot 0.61712 \approx 25.619 \pm 1.423$$

$$b = 184.074 \pm 2.306 \cdot 257.824 \approx 184.07 \pm 594.54$$

$$k = 0.11929 \pm 2.306 \cdot 0.21319 \approx 0.119 \pm 0.049$$

$$c = -0.921622 \pm 2.306 \cdot 0.331366 \approx -0.922 \pm 0.764$$

ERRORS IN CLASSICAL PLANT GROWTH ANALYSIS

Classical plant growth analysis is of great value for botanists and physiologists. It consists in determination of dry matter and leaf area changes during ontogenesis and in calculation of parameters (being a combination of the measured quantities), which permit inquiring into the basic physiological processes occurring in a plant. Taking at least two measurements of dry matter and assimilating surface of plants during the ontogenesis, the most frequently used growth characteristic may be calculated using the interval method (Kvet et al. 1971):

Growth rate (GR):

$$GR = \frac{W_2 - W_1}{t_2 - t_1} \quad [\text{weight time}^{-1}]. \quad (19)$$

Relative growth rate (RGR):

$$RGR = \frac{\ln W_2 - \ln W_1}{t_2 - t_1} \quad [\text{weight weight}^{-1} \text{ time}^{-1}]. \quad (20)$$

Unit leaf rate (ULR):

$$ULR = \frac{W_2 - W_1}{t_2 - t_1} \cdot \frac{\ln A_2 - \ln A_1}{A_2 - A_1} \quad [\text{weight area}^{-1} \text{ time}^{-1}] \quad (21)$$

(on condition that there is a linear dependence between A and W).

Leaf area ratio (LAR):

$$LAR = \frac{A_2 - A_1}{\ln A_2 - \ln A_1} \cdot \frac{\ln W_2 - \ln W_1}{W_2 - W_1} \quad [\text{area weight}^{-1}] \quad (22)$$

$$\text{or} \quad LAR = \frac{RGR}{ULR} \quad (23)$$

In the above equations the symbols W_1 , W_2 and A_1 , A_2 represent biomass W (measured usually as dry matter) and assimilatory surface area A (measured usually as a leaf surface area), recorded at two subsequent harvests made at times t_1 and t_2 . We assume, that errors originated from dry matter measurement and leaf surface area are random errors, resulting from individual variation of the studied genotype. That is why mean values W and A from several replications should be taken for calculations, and basing on Student's-t distribution the estimated confidence intervals for those random variables ($\pm \Delta W$, $\pm \Delta A$) should be reckoned.

Because growth characteristics are a function of at least two variables (from which every one is defined with a certain absolute error), the total differential method should be used for calculations, thanks to which one may define the error of N-argumented function Δy , generated by the values of arguments x_i :

$$\Delta y = \sum_{i=1}^N \left| \frac{\partial f}{\partial x_i} \right| \Delta x_i \quad (24)$$

On the basis of the above equation absolute errors of the analysed growth characteristics are described as follows:

$$\Delta GR = \frac{\Delta W_2 - \Delta W_1}{t_2 - t_1} \quad (25)$$

$$\Delta RGR = \frac{1}{t_2 - t_1} \left(\frac{\Delta W_2}{W_2} + \frac{\Delta W_1}{W_1} \right) \quad (26)$$

$$\begin{aligned} \Delta ULR = & \frac{\ln A_2 - \ln A_1}{(t_2 - t_1)(A_2 - A_1)} (\Delta W_2 + \Delta W_1) + \\ & + \frac{|W_2 - W_1|}{(t_2 - t_1)(A_2 - A_1)^2} \left[\left(1 - \frac{A_1}{A_2} \right) - \ln \frac{A_2}{A_1} \right] \Delta A_2 + \\ & + \left[\left(\frac{A_2}{A_1} - 1 \right) - \ln \frac{A_2}{A_1} \right] \Delta A_1 \end{aligned} \quad (27)$$

$$\Delta LAR = LAR \cdot \left(\frac{\Delta RGR}{|RGR|} + \frac{\Delta ULR}{|ULR|} \right) \quad (28)$$

For example of calculations my own data concerning growth of maize plant (*Zea mays* L.) were used. Plant measurements were done at 6 stages of development. Mean values of dry matter of the overground part and leaf blade area from 5 replications were taken for calculations. Using tables of Student's-t distribution the estimation of mean interval value has also been done for significance level $\alpha = 0.05$. All data are compiled in Table 2. Below absolute errors have been calcu-

TABLE 2. Mean values of dry matter and leaf blade area in 6 development stages of maize (*Zea mays* L.).

Measurement	Development stage	Time, t (day)	Dry matter $W \pm \Delta W$ (g)	Leaf area $A \pm \Delta A$ (dm ²)
1	5 leaf	48	4.01 $\pm$ 0.55	2.57 $\pm$ 0.33
2	7-8 leaf	66	28.38 $\pm$ 6.89	17.77 $\pm$ 1.16
3	Tasseling	78	57.61 $\pm$ 6.54	37.59 $\pm$ 4.00
4	Blister maturity	93	100.56 $\pm$ 8.51	35.40 $\pm$ 3.03
5	Dough maturity	121	135.58 $\pm$ 14.42	36.96 $\pm$ 6.15
6	Physiological maturity	139	141.54 $\pm$ 24.03	—

ΔW , ΔA – confidence limits (standard error $\times$ Student's-t)

lated – as an example – for the period from tasseling stage till blister maturity stage.

$$\Delta GR = \frac{8.51 + 6.54}{93 - 78} = 1.003 \text{ g day}^{-1}$$

$$\Delta RGR = \frac{1}{93 - 78} \left(\frac{8.51}{100.56} + \frac{6.54}{57.61} \right) = 1.321 \cdot 10^{-2} \text{ g g}^{-1} \text{ day}^{-1}$$

$$\begin{aligned} \Delta ULR = & \frac{\ln(35.40/37.59)}{(93 - 78)(35.40 - 37.59)} (8.51 + 6.54) + \\ & + \frac{|100.56 - 57.61|}{(93 - 78)(35.40 - 37.59)^2} \left[\left(1 - \frac{37.59}{35.40} \right) - \ln \frac{35.40}{37.59} \right] 3.03 + \\ & + \left[\left(\frac{35.40}{37.59} - 1 \right) - \ln \frac{35.40}{37.59} \right] 4.00 = 3.504 \cdot 10^{-2} \text{ g dm}^{-2} \text{ day}^{-1} \end{aligned}$$

$$\Delta LAR = 0.473 \left(\frac{1.321 \cdot 10^{-2}}{3.714 \cdot 10^{-2}} + \frac{3.504 \cdot 10^{-2}}{7.848 \cdot 10^{-2}} \right) = 0.379 \text{ dm}^2 \text{ g}^{-1}$$

Results of the above calculations with relative errors and with the values of maize plant growth characteristics for that vegetation period are presented in Table 3.

TABLE 3. Values and errors of characteristic calculations of maize productivity (*Zea mays* L.) in period from development stage of tasseling till blister maturity

Growth characteristic	Unit	Value	Absolute error	Relative error (%)
GR	g day ⁻¹	2.863	1.003	35.0
RGR	g g ⁻¹ day ⁻¹	3.714 $\cdot$ 10 ⁻²	1.321 $\cdot$ 10 ⁻²	35.6
ULR	g dm ⁻² day ⁻¹	7.848 $\cdot$ 10 ⁻²	3.504 $\cdot$ 10 ⁻²	44.6
LAR	dm ² g ⁻¹	0.473	0.379	80.1

CONCLUSIONS

In the present work the error estimation has been performed, which may occur in calculation of plant growth analysis. In quantitative growth analysis (functional analysis) the measure of estimation accuracy for dry matter values was the standard error and the build on it confidence interval of the expected value of that random variable. A relatively wide

confidence interval results from necessity of linearization of a growth function by means of logarithmic transformation and from a small number of measurements. In case of several variables functions (e. g. Richard's function) variance of dependent variable is a consequence of error propagation law. In Richards function (equation 15) particularly the b coefficient has been estimated with a considerable standard error. In such a case the statistical verification of significance of estimated coefficients is recommended and eventually acceptance of another regression function. There is also a considerable contribution of coefficient k in variance s_y^2 (equation 16). It results from exponential function propriety, in which relatively small changes of the argument cause large changes of value of the dependent variable. It should be remembered that in numerical methods the starting values of coefficients chosen by us influence the values of the searched parameters.

The considerable absolute errors of parameters in classical growth analysis result from two main reasons. Firstly, plant genotype does not determine the final morphological form, but only the limits of phenotype variability, which practically causes great individual variability of the examined plants. Secondly, in case of several variables functions, theorems of error calculus assume uniformly cumulating partial errors originating from different arguments. The probability of extremely unfavourable error cumulation decreases markedly with the increase of argument number and practically such a case occurs occasionally. It should be remembered that in the expansion of Taylor series (equation 9) the terms were neglected, in which the argument error occurs in a power greater than first and that than great errors of independent variables cause simultaneously considerable inaccuracy of error estimation of a growth characteristic.

In calculations of plant growth analysis estimation of committed errors is seldom done (Causton and Venus 1987). This procedure should, however, not be neglected because only the

combination of error calculus with statistical methods gives a complete picture of the studied phenomenon.

LITERATURE CITED

- BLISS C.I., 1970. Statistics in biology, vol. 2. McGraw – Hill Book Company. London.
- CAUSTON D.R., VENUS J.C., 1987. The biometry of plant growth. E. Arnold. London.
- CHATTERJEE S., PRICE B., 1977. Regression analysis by example. J. Wiley and Sons. New York.
- DRAPER N.R., SMITH H., 1973. Applied regression analysis. J. Wiley and Sons. New York.
- EVANS G.C., 1972. The quantitative analysis of plant growth. Blackwell Scientific Publications. Oxford.
- GREGORCZYK A., 1991. The logistic function – its application to the description and prognosis of plant growth. Acta Soc. Bot. Pol. 60: 67-76.
- GREGORCZYK A., 1994. Application of the Richards function to the description of leaf area growth in maize (*Zea mays* L.). Acta Soc. Bot. Pol. 63: 5-7.
- HUNT R., 1982. Plant growth curves. E. Arnold. London.
- KORN G.A., KORN T.M., 1968. Mathematical handbook for scientists and engineers. McGraw – Hill Book Company. New York.
- KVET J., ONDOK J.P., NECAS J., JARVIS P.G., 1971. Methods of growth analysis. In: Plant photosynthetic production. Sestak Z. (ed.). Dr W. Junk N. V. Publishers, The Hague, pp. 343-391.
- SNEDECOR G.W., COCHRAN W.G., 1968. Statistical methods. The Iowa State University Press. Ames.
- STEEL R.D.G., TORRIE J.H., 1987. Principles and procedures of statistics. Mc Graw – Hill Book Company. Auckland.
- WEISBERG S., 1985. Applied linear regression. J. Wiley and Sons. New York.
- ŻELAWSKI W., LECH A., 1980. Logistic growth functions and their applicability for characterizing dry matter accumulation in plants. Acta Physiol. Plant. 2: 187-194.

ESTYMACJA BŁĘDÓW W ANALIZIE WZROSTU ROŚLIN

STRESZCZENIE

Przedstawiono schemat obliczeń błędów wartości suchej masy przy aproksymacji danych krzywymi wzrostu, wyznaczonymi metodami analitycznymi (funkcja logistyczna) i numerycznymi (funkcja Richardsa). Zamieszczono także wzory, określające błędy bezwzględne wskaźników wzrostu: absolutnej szybkości wzrostu (GR), względnej szybkości wzrostu (RGR), jednostkowej produktywności liści (ULR) i wskaźnika ulistnienia (LAR). Przytoczono przykłady rachunkowe, dotyczące przebiegu wzrostu roślin owsa i kukurydzy. Dokonano krytycznej analizy otrzymanych oszacowań wyników. Stwierdzono celowość łącznego stosowania w analizie wzrostu roślin metod statystycznych i rachunku błędów.

SŁOWA KLUCZOWE: analiza wzrostu roślin, akumulacja suchej masy, rachunek błędów, funkcje regresji, *Zea mays* L., *Avena sativa* L.