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A mathematical description of maize leaf area growth using a logistic curve

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Abstract

In this paper, an attempt is made to apply the Verhulst-Pearl and the Robertson logistic curves to the description of maize plant growth. The changes with time of the total leaf area were taken as the parameter expressing the growth kinetics. The constant coefficients in the Verhulst-Pearl and Robertson equations were calculated with the help of a logarithmic transformation and the least square method. On this basis, the growth kinetics of the studied maize lines and hybrid were compared. The applicability of logistic curves to the description of completed growth processes was demonstrated as was the fact that Robertson's equation is better suited for mathematical calculations.

INTRODUCTION

The problem of plant growth has for a long time been receiving the attention of biologists, who have been trying to find an universal mathematical formula describing this physiological process. This problem remains, however, still unresolved due to the very complicated mechanism of ontogenesis which encompasses numerous complex biochemical and biophysical processes. One of the most outstanding difficulties here is the exact determination of the influence of environmental factors which modify the growth of plants to a great extent. It seems that none of the numerous mathematical formulas found in literature (Sztencel and Żelawski, 1984) can be regarded as an universal function serving to forecast the growth of organisms. In addition, some of the functions, due to their complicated shape, have limtied practical application.

It is, however, purposeful to use certain mathematical equations to analyse completed growth processes (Żelawski and Lech, 1980). Growth curves are presented as the function of time of a specified dimension or of the organism's mass. These curves are, for many plant and animal species (and even for their parts and organs) usually similar in shape to the letter S. One of the commonly used functions having this shape is the so-called logistic curve. This functions is the solution to the Verhulst-Pearl equation (Glaser, 1975) or the Robertson equation (Salisbury and Ross, 1975). The Verhulst-Pearl equation is as follows:

$$\frac{dy}{dt} = (k_1 - k_2 y)\, y = k_1 y - k_2 y^2, \tag{1}$$

where: y — magnitude of growth
t — time
k_1 — a coefficient characterizing the intensity of anabolism
k_2 — a coefficient characterizing the intensity of catabolism.
After integrating the equation, the following dependency $y(t)$ is obtained:

$$y = \frac{y_0 k_1 e^{k_1 t}}{k_1 + k_2 y_0 (e^{k_1 t} - 1)} \qquad y_0 = y(0). \tag{2}$$

Robertson holds that the growth of plants runs analogically to the rate of autocatalytic reactions, that is, that the growth rate is proportional to the current value expressing growth and to the difference between its starting and current values.

$$\frac{dy}{dt} = ky\,(y_{max} - y) \tag{3}$$

k — constant growth rate
y_{max} — asymptotic, maximum growth magnitude.
After solving equation (3), we obtain:

$$y = \frac{y_{max}}{1 + e\,[^{-}y_{max}\, k\, (t - t_{1/2})]} = \frac{y_{max}}{1 + \exp\,[^{-}y_{max}\, k\, (t - t_{1/2})]}. \tag{4}$$

$y(t_{1/2}) = {}^1/_2\, y_{max}$.

The constant $t_{1/2}$, the time during which one-half of the maximum growth magnitude is attained, (${}^1/_2 y_{max}$), was introduced in the process of integration.

Functions (2) and (4) are solutions of the same differential equation — they differ only by the coefficients which have been accepted.

MATERIAL AND METHOD

The pot experiment was conducted in 1984 in the vegetation hall of the Szczecin Academy of Agriculture. Two strains of maize were used, EP1 and SR10 and their hybrid, EP1×SR10.

The parameter used for expressing the growth kinetics was the change in time of the total area of green, developed leaf blades, since maize leafs are an important part of the plant and due to the efficiency of photosynthesis, determine the yield (Andrejenko and Kuperman, 1962).

The biometrical measurements were made every 7 days when the length and the greatest width of the leaf blades were recorded. Next, the leaf area was calculated using a constant coefficient (Strickler, 1961). The measurements were concluded when the studied plants attained the maximum total leaf-blade area, which took place during the blooming phase. The mean values of 5 of the same plants were used in the mathematical calculations.

Robertson's equation was used to find specific forms of the logistic function. After transformation of this function (by taking logarithms) (Bliss, 1970), the following was obtained:

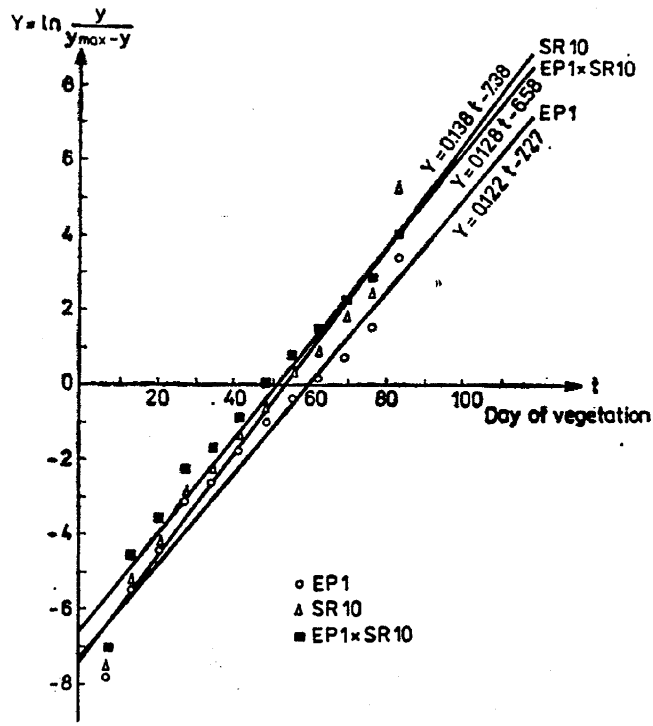

Fig. 1. Linear regression functions $Y = B \cdot t + A$ for the investigated maize forms

$$\ln \frac{y}{y_{max} - y} = y_{max}\, k\,(t - t_{1/2}) = y_{max}\, k\, t - y_{max}\, k\, t_{1/2} \tag{5}$$

which can be reduced to a linear function.

$$Y = B \cdot t + A. \tag{6}$$

Next, by the least square method, the coefficients A and B were determined, and from them, the values of the constants: k, $t_{1/2}$ and k_1, k_2 and y_0. For each breeding form, the determination coefficient r^2 was also calculated. The coefficient expresses which part of the total variability of the dependent variable is explained by the independent variable, t. On Figure 1, the linear regression function, $Y = B \cdot t + A$ is presented for the studied plants.

RESULTS AND DISCUSSION

The detailed forms of the logistic function according to Robertson and Verhulst-Pearl are given in Table 1.

Table 1

Growth equations of the studied maize plants

Breeding form	Growth equation (author)	
	Robertson	Verhulst-Pearl
EP1	$y = \frac{2408}{1+\exp[-0.122\,(t-59.6)]}$	$y = \frac{0.204\ \exp\,(0.122t)}{0.122+8.47\cdot10^{-5}[\exp\,(0.122t)-1]}$
SR10	$y = \frac{2595}{1+\exp[-0.138\,(t-53.5)]}$	$y = \frac{0.224\ \exp\,(0.138t)}{0.138+8.62\cdot10^{-5}[\exp\,(0.138t)-1]}$
EP1 × SR10	$y = \frac{3628}{1+\exp[-0.128\,(t-51.4)]}$	$y = \frac{0.644\ \exp\,(0.128t)}{0.128+17.8\cdot10^{-5}[\exp\,(0.128t)-1]}$

In order to compare the growth patterns of the three studied breeding forms, the coefficients characterizing growth equations are compiled in Table 2. The greatest total leaf blade area was attained by the hybrid, EP1×SR10, the remaining lines were characterized by similar, although significantly lower $y_{max.}$ values. The leaf areas found in this experiment fit in the lower limit of the values found for maize plants (Andrejenko and Kuperman, 1962). The coefficients k_1 (day^{-1}), which express the intensity of anabolic processes, are significantly higher than the coefficients k_2 (cm^{-2} day^{-1}), which characterize catabolic processes.

Table 2

Growth equation coefficients

Breeding form	y_{max} cm^2	$k=k_2$ cm^{-2} day^{-1}	$t_{1/2}$ day	k_1 day^{-1}	y_0 cm^2	$(dy/dt)_{max}$ cm^2 day^{-1}	r^2
EP1	2408	$5.07 \cdot 10^{-5}$	59.6	0.122	1.67	73	0.963
SR10	2595	$5.32 \cdot 10^{-5}$	53.5	0.138	1.62	90	0.976
EP1 × SR10	3628	$3.53 \cdot 10^{-5}$	51.4	0.128	5.03	116	0.973

This is understandable, since according to the Verhulst-Pearl equation, anabolism is proportional to the magnitude of growth to the first power, and catabolism, to the growth magnitude squared. The k_2 coefficients are similar for both lines, whereas for the hybrid, the k_2 value is decidedly lower. This explains the quicker growth of hybrid EP1×SR10 in comparison with its parental lines. This hybrid attained one-half of its maximum leaf area more quickly, which is shown by the lower $t_{1/2}$ (day) value. The greater value of coefficient k_1 for the SR10 form in comparison with EP1 plants (with similar k_2 values) proves the somewhat quicker growth of the maternal line. This is also supported by the half-period ($t_{1/2}$) values.

The theoretical y_0 value is the highest for the hybrid (it is difficult to obtain significant differences for this line). In the exponential phase of growth, the y_0 value has a decisive influence on the values of y obtained, which results from the limit:

$$\lim_{t \to 0} \frac{y_0 k_1 e^{k_1 t}}{k_1 + k_2 y_0 (e^{k_1 t} - 1)} = y_0 \tag{7}$$

When the second derivative of the function $y(t)$ is calculated and then compared to zero, the time is obtained at which the curve reaches its point of inflection and the growth rate attains its maximum; this is the value of the half-period $t_{1/2}$. At this time, the maximum growth rate is equal to:

$$\left(\frac{dy}{dt}\right)_{max} = 1/4\, k\, y^2_{max}. \tag{8}$$

The greatest theoretical growth rate (116 cm^{-2} day^{-1}) is characteristic for the hybrid, the lowest (73 cm^{-2} day^{-1}) for form EP1.

The growth curves for the analysed maize plants are presented on Figure 2.

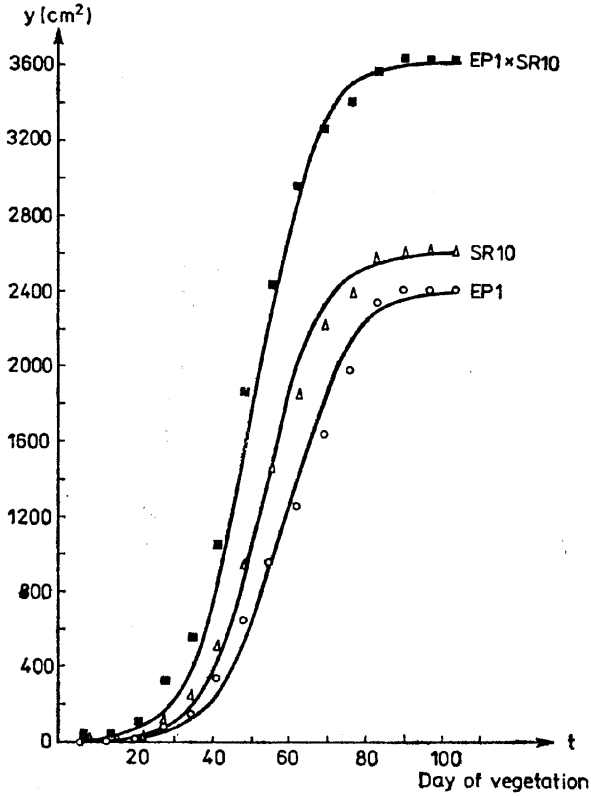

Fig. 2. Theoretical growth curves of maize plants

CONCLUSIONS

The following conclusions may be drawn on the basis of the conducted experiments and calculated results:

1) The application of logistic curves to the mathematical description of completed growth processes is purposeful. This is shown by the high determination coefficients, r^2, albeit, the limitations introduced by the logarithmic transformation should be taken into account.

2) Both forms of the logistic curve are useful in the interpretation of the experimental results, although Robertson's equation is better suited for mathematical calculations.

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Matematyczny opis wzrostu powierzchni liści roślin kukurydzy za pomocą funkcji logistycznej

Streszczenie

W pracy podjęto próbę zastosowania funkcji logistycznej według Verhulsta-Pearla i Robertsona do opisu wzrostu roślin kukurydzy. Jako wielkość określającą kinetykę wzrostu przyjęto zmiany w czasie sumarycznej powierzchni liści. Za pomocą transformacji logarytmicznej i metodą najmniejszych kwadratów obliczono stałe współczynniki do równań Verhulsta-Pearla i Robertsona. Na tej podstawie dokonano porównania kinetyki wzrostu badanych linii i mieszańca kukurydzy. Stwierdzono celowość stosowania funkcji logistycznej do opisu zakończonych już procesów wzrostu, przy czym do obliczeń matematycznych lepiej nadaje się równanie Robertsona.